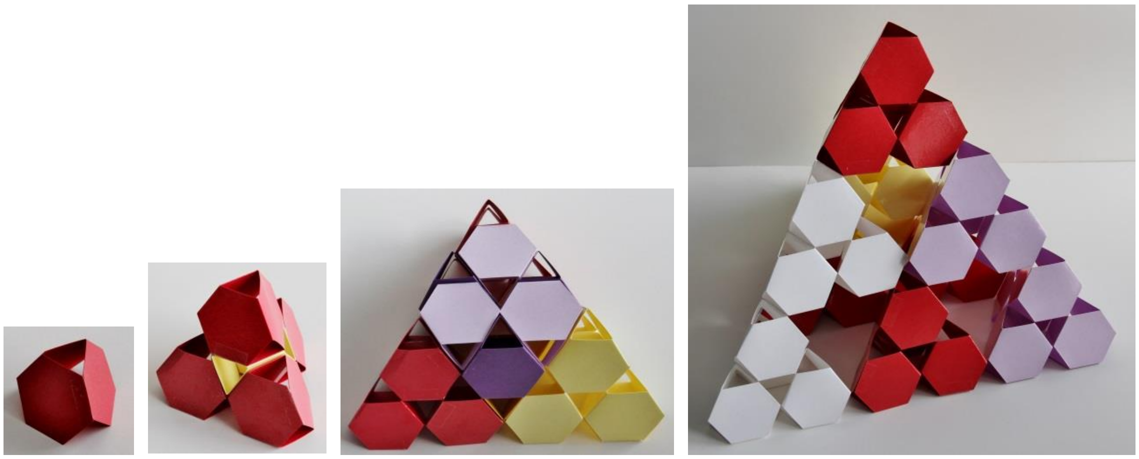
Workshop: XColony – Advanced Series – Sierpinski Pyramids

Sorin Alexe, Ph.D., Jan 28, 2015



Sierpinski Pyramids: from basic module to the full model

The activities described in this document should be scheduled across several sessions (about 8 hours). Each session can follow the flow defined below and should reach the partial objectives, such that all goals are fulfilled by the end of the last session.

Objectives

Learn advanced properties of truncated tetrahedra. Understand the self-similarity structures and fractal-like features. Understand volume and spatial tessellations Forecast, extrapolated and discover spatial structure of increasing complexity. Learn how to collaborate, coordinate and distribute tasks. Analyze the benefits of team work. Use all intermediary steps to pose new problems, and investigate new properties.

Use the KDK components to solve various puzzles and to build the constructions described in the accompanying manual.

Materials

Movies from the XColony channel on YouTube and KDK DVD.

Elements and documents from the KDK box
Internet resources: pictures, documents, movies

Organization

The workshop is organized in groups of 4-6 participants.

Activities

1. Presentation of the theme and objectives (10 min)

- watch the introductory and step-by-step instructions movies
- brainstorming on the content of the movies: how many items have been identified, what kind of description could be associated with them, what other names should be given to these items in order to better designate them? Find associations with objects from reality and life, science (flowers, animals, crystals, molecules) and art (films, tilings, architecture, origami).

2. Distribution of materials (5 min)

- identification/designation of the elements,
- generate hypothesis on their use and foresee the final construction to be made

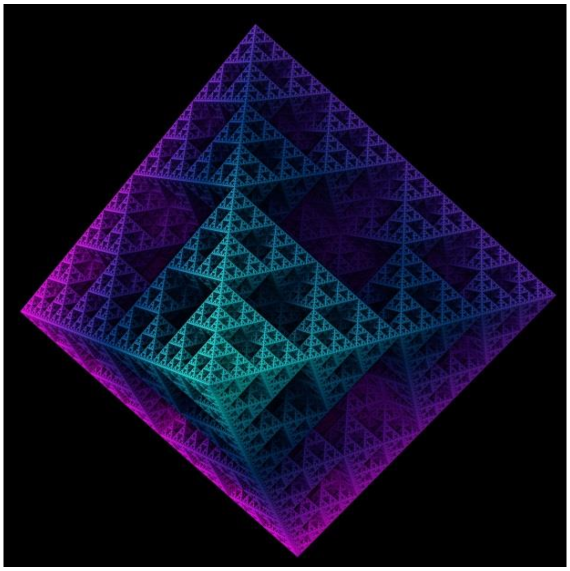
3. Playing and Learning Activities. Construction, Puzzle solving, Discovery (40 min)

Progress with activities specified in Appendix. Each session should advance to the degree that the last session will conclude this module. If the material is too extensive relative to students' development and skills, some parts may be reduced or skipped; the focus should be on the constructions.

- each group will inspect the materials and investigate through puzzle solving the 2D properties of the available elements
- visualize the way to construct the basic modules and build all of them
- formulate the strategy to be applied to obtain the final object: schedule tasks, assign tasks to members of the group equitably, proceed
- investigate geometric, kinetic and combinatorial properties of the final construction; visit other groups and discuss your findings; ask questions, challenge the others to pose problems and puzzles, ask for new solutions.

4. Concluding the workshop (5 min):

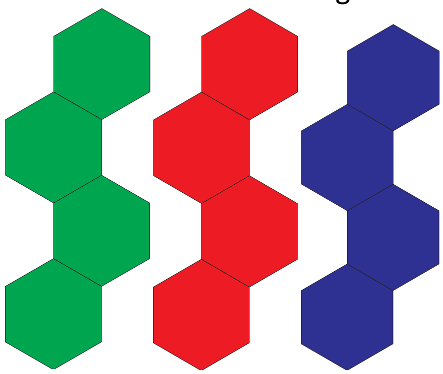
- recall the most interesting part of the workshop (each group identify one)
- pack and store or distribute the final constructions made. If the constructions are distributed there should be a fair system for that (e.g., a number of credits is assigned for every participant, each construction costs a number of credits, the student with a highest number of credits in the group will pick first and his/her number of credits is adjusted by the cost of the object). Students should be encouraged to design their own system of credits). After several sessions all students would have taken home at least one object.



Appendix

Activity 1

Inspect the 200 Hs4 elements having different colors.



Position some of them in plane such that you get configurations with the following shadows:

Create 2 more problems of this type.

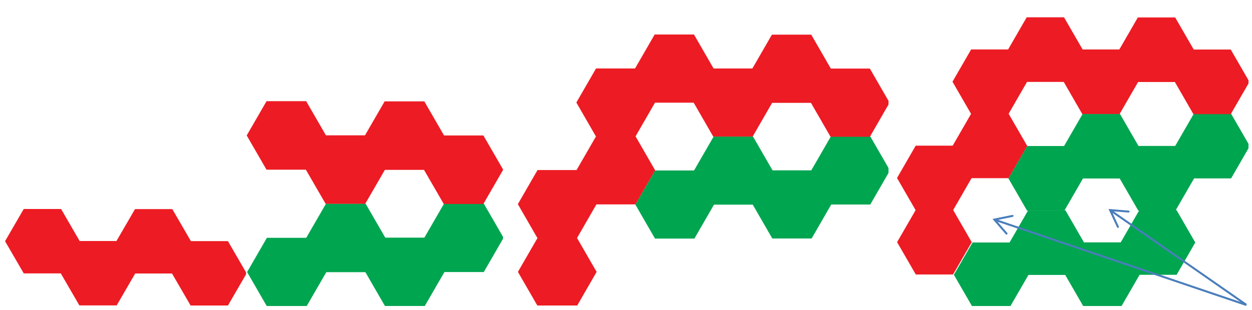
Activity 2

Define an area by joining blue Hs4 elements. Create a boundary for this area using elements of different colors, say yellow and orange, without overlapping. How many elements do you need? What is the minimum number? What about the maximum number? Examples of such activities with 1,2 and 3 blue elements are shown below.

Can you border the 2 blue elements using only 4 elements? Yes, but the blue area needs to be redefined.

Activity 3

Play with one of your colleagues. The first configuration starts with 1 element on the table. Players has to take turns and place one element such that at least one hole is created. The player that creates 2 or more holes in one move wins the game.



Green player wins

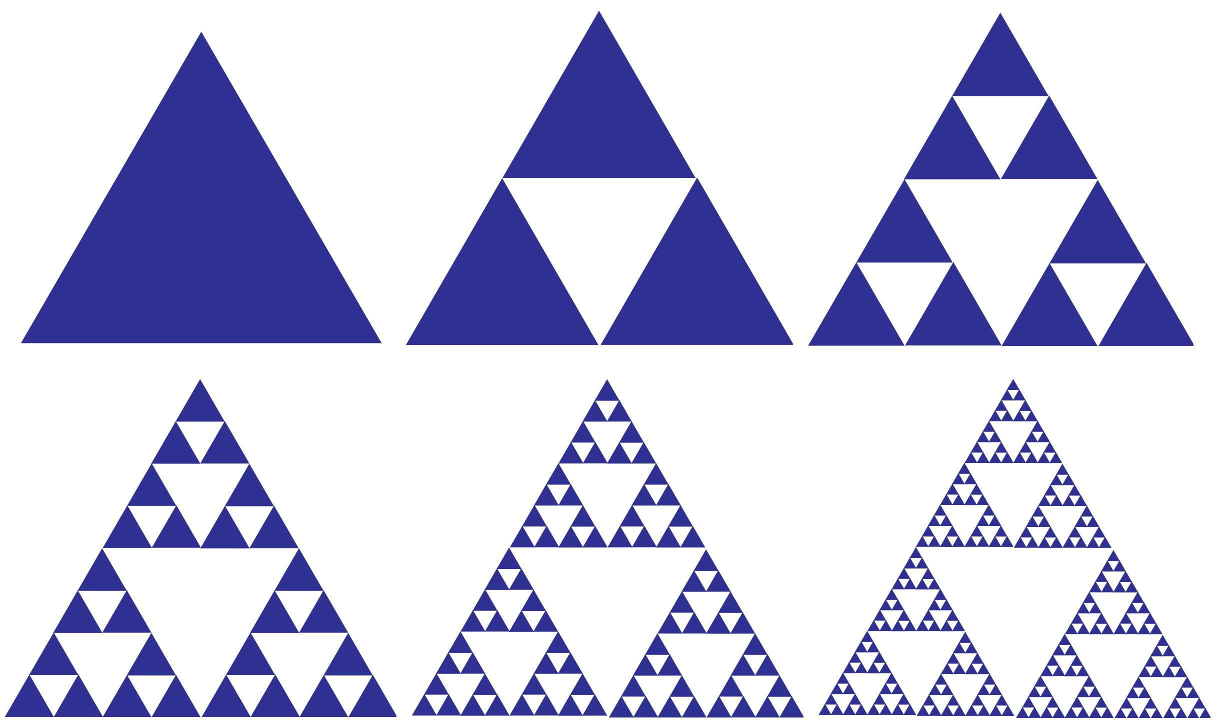
Activity 4

The Sierpinski triangle is a 2D object obtained in the following way:

1. Start with a triangle
2. Mark on each of its sides the middle point. Connect the three middle point of the triangle and remove the small triangle in the middle.
3. Repeat step 2 indefinitely

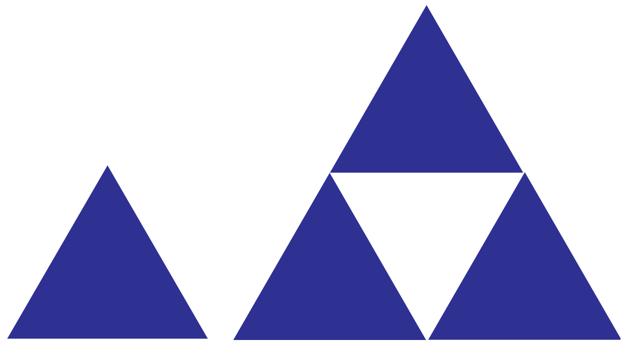
The Sierpinski triangle is a fractal, i.e., a self-similarity structure: it has the same aspect no matter how close you are looking at it.

Here you have a sequence of pictures showing the first 5 iterations. How many triangle can you identify at each step? After how many iterations do you get 17 triangles?

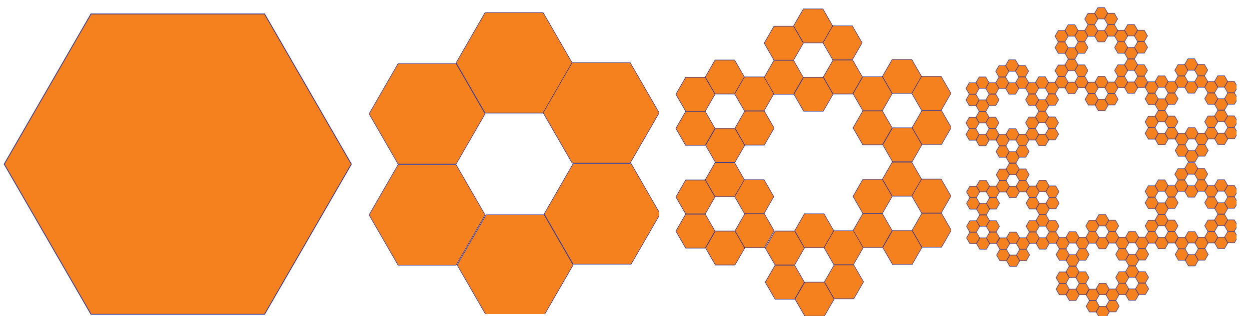


How is the blue area changes from one step to the other?

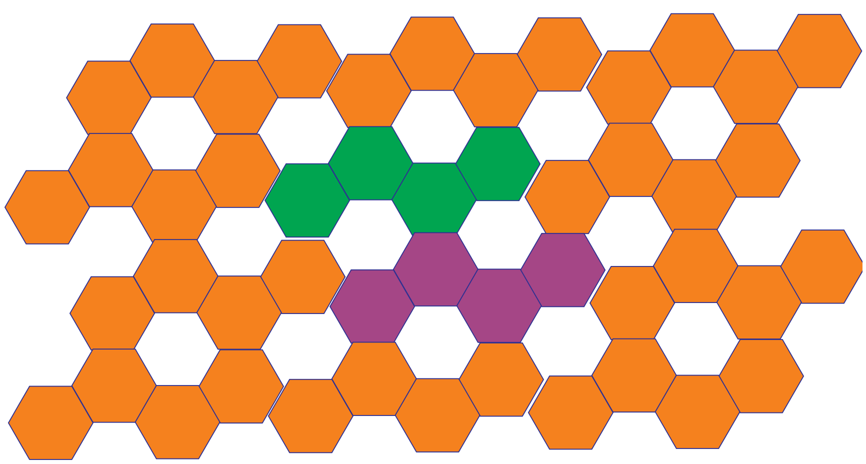
Define a Sierpinski process in a constructive way: “build and scale down”: Start with one triangle, make 3 copies (for each of its vertices) and use its structure to join the copies. Shrink the final construction to same size as the original object.



Can you identify a Sierpinski process with hexagons?



The regular hexagon will allow you to construct a tiling of the plane. Can you make a tiling with the other objects in the sequence? Is it possible to replicate the one associate to the first itetration using the Hs4 elements from the KDK?

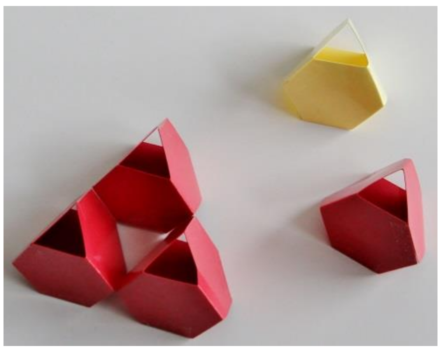
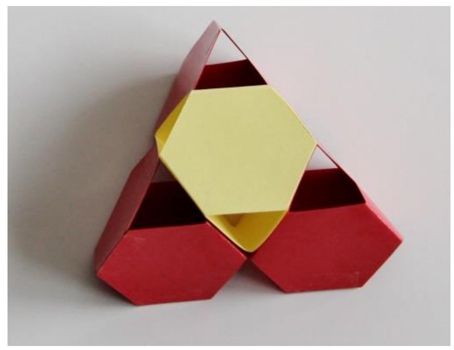


Activity 5

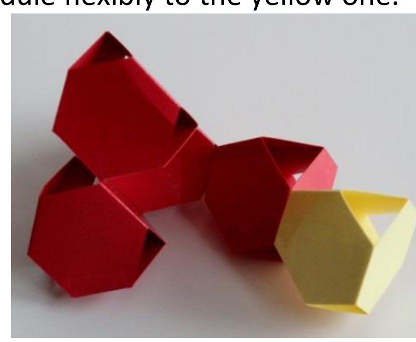
Imagine a regular tetrahedron and cut its corners such that all faces become regular hexagons. Is it possible?

Can you visualize the Sierpinski pyramids?

Pick 4 elements Hs4 of the same color (say red) and one of a different color (say yellow). Construct the T modules by linking 3 pairs of edges. Link 3 red module such that the yellow one can sit in the middle.

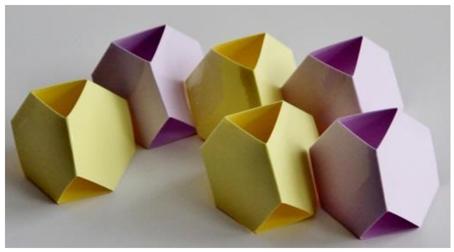
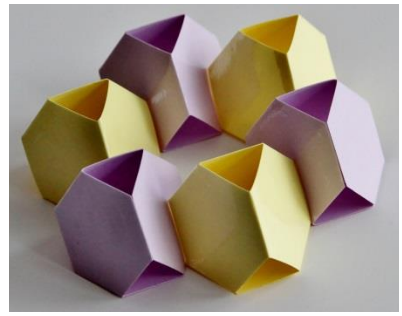
Link flexibly the 4th red T module. Link this module flexibly to the yellow one.

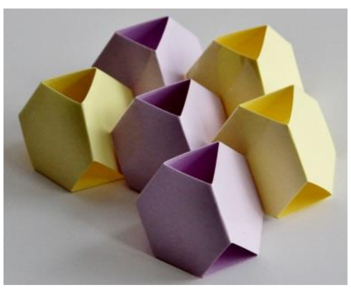
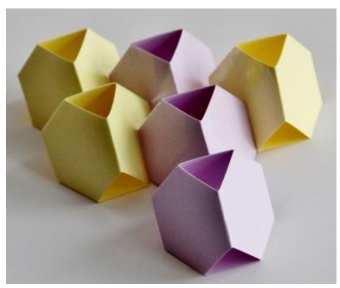
Now you can hide the yellow module inside the red structure.

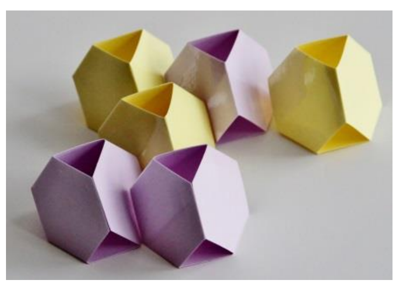

Activity 6

Assemble the 6 T modules, in 2 colors (3 modules each). Arrange them in the configuration shown below, on the left. You can move any module (only one at a time) such that in its new position it touches 2 modules (see middle). Repeating this type of moves you have to obtain the configuration depicted in the right picture.

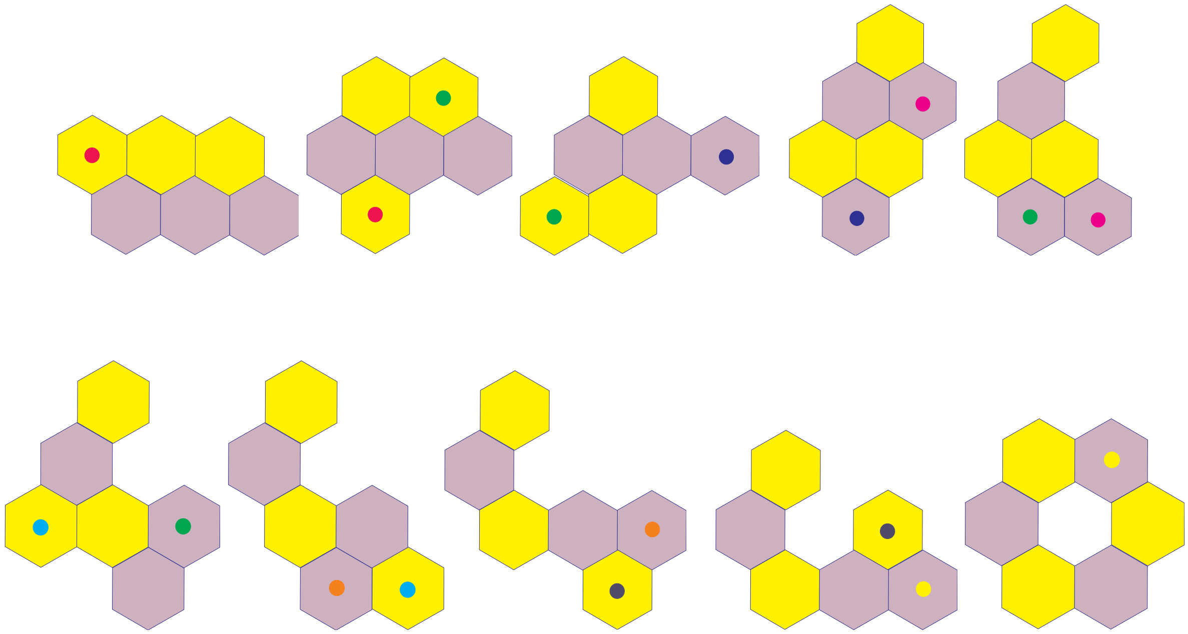




Here is one possible solution:

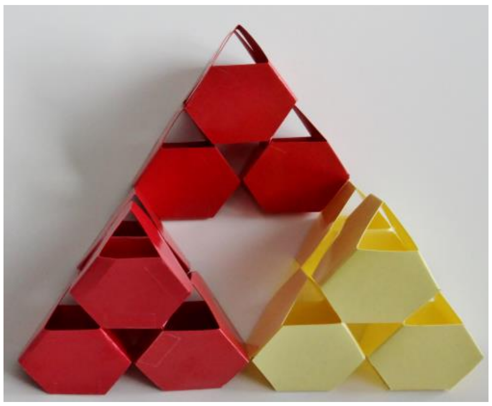

The footprint of the bases in 2D is presented schematically:



Activity 7

Assemble as many T modules as you like, even all of them. Using 4 T modules you can build a T.2 module. This is a tetrahedron whose vertices are instantiated by T modules. The construction is similar to the one in Activity 5, but this time close all connections to obtain a better stability. The T.2 module has a fractal property: it resembles the T module and it is built from T modules. Can you visualize the next step T.2.2? How would a T.2.3 look like? Yes, it will be a “tetrahedron” with 3 T.2 modules on each edge.

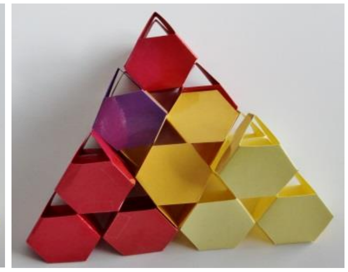
Join 3 T2 modules as shown below. (This is also an idea on how to think about T.2.2)



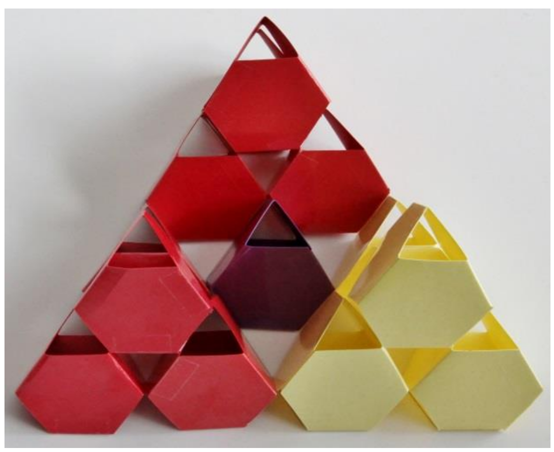
Inspect the interior of this construction. How many T modules can you fit inside?

Say you start placing T modules inside the structure. You might be able to observe a sequence like the next one:



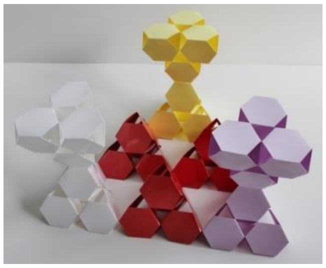


If your answer was 9 then you missed the very first one (see picture below).

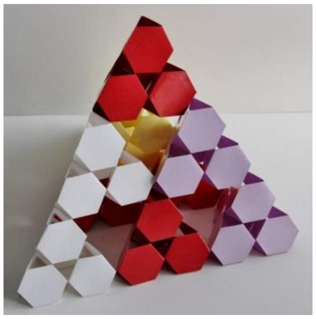


Activity 8

Start with eight T.2 and connect thenm flexibly so that the construction can take the following shapes:



Use the ninth T.2 module to complete the conctruction to T.2.3.



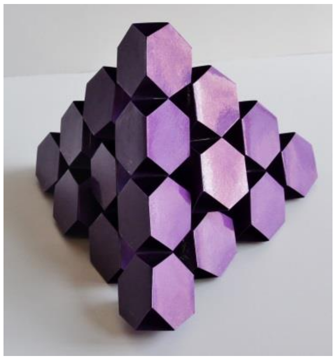
Activity 9

This activity is not intended to be hands-on. However, in a multiple-group workshop the ideas can be tried out and investigated further.

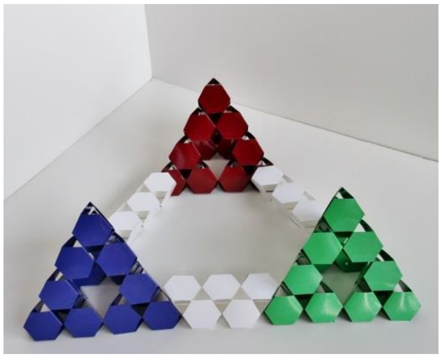
What would a T.2.3.2 look like? Yes, this is a structure obtained by using four T.2.3 modules like the one obtained in Activity 8 and linking them following the structure of a T.2 module. How many T modules would be needed?

Explore this problem for different configurations:

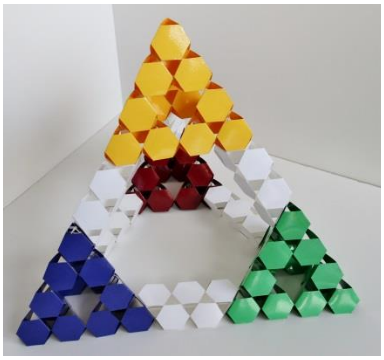
T.3.2, T.2.2.2, T.2.2.2.2, T.3.3.3?



T.4



Three T.2.2 Connected



Expanded T.2.2.2